

$$2\pi f$$

$$= \omega^2 r$$

$$= m\omega^2 r$$

data booklet!

Periodic motion

Simple harmonic motion

1

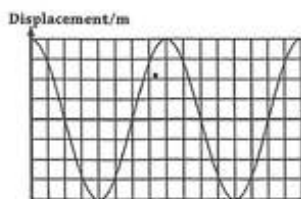
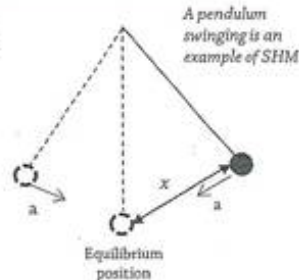
Simple harmonic motion is any oscillating motion whose acceleration is **proportional to its displacement** and **opposing in direction to the displacement**.

The essential condition for simple harmonic motion is:

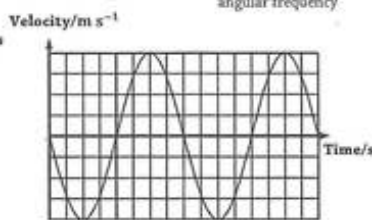
$$a \propto -x$$

$$a = -\omega^2 x$$

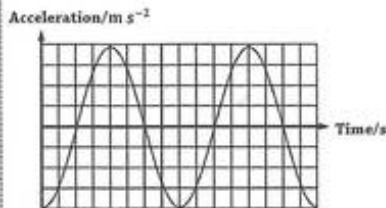
a is acceleration, x is displacement and ω is angular frequency



$$\text{displacement} = x = A \cos \omega t$$



$$\text{velocity} = v = A \sin \omega t$$



$$\text{acceleration} = a = -A \cos \omega t$$

- The $v-t$ graph can be found from the **gradient** of the **displacement-time** graph.
- The $a-t$ graph can be found from the **gradient** of the **velocity-time** graph.

Exam Tip!

The minus sign in the simple harmonic equation indicates the acceleration is in the opposite direction to the displacement.

Velocity can also be determined from:

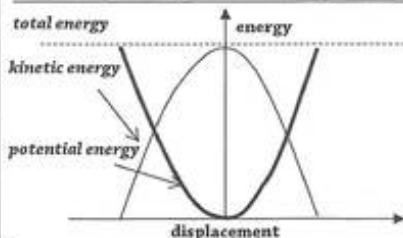
$$v = \pm \omega \sqrt{A^2 - x^2}$$

Since the object will be travelling fastest when $x = 0$,

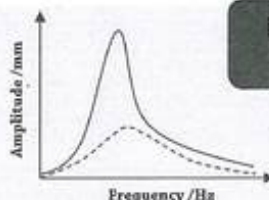
$$v_{\text{max}} = \omega A$$

Maximum velocity occurs at minimum displacement, or when $x = 0$.

Energy-displacement graphs of SHM for free oscillations



The **total energy** of a simple harmonic system is constant. At minimum displacement, speed is at a maximum and so **kinetic energy** is maximum. Kinetic energy is converted to **potential energy** as displacement increases.



Forced vibrations and resonance

Resonance: the situation when the periodic force driving the forced oscillation is **in phase** with the frequency of the oscillating system.

Think of a child on a swing. Typically, stronger pushes gives a larger swing, but at a specific frequency, the swinging increases greatly and becomes erratic.

Light damping:

- The resonant frequency (frequency at maximum amplitude) will be **the same as the natural frequency**
- The maximum amplitude at resonance will be greater

Heavier damping:

- The **amplitude** at resonance **reduces**
- Frequency** at which resonance occurs **decreases**