

Fields

Force field: a region in which a body experiences a non-contact force. It can be represented by a vector.

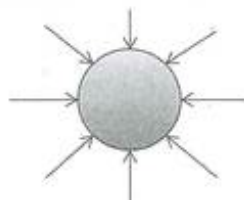
Force fields can arise from:

- Interactions of masses
- Interactions of static charge
- Interactions between moving charges



Gravitational fields

Radial field: A field that is spherical and acts towards a central point. Gravitational fields are radial. A radial gravitational field around a planet can be seen to the right.



Uniform field: A field that has no variation. Gravitational fields are approximated as uniform near the surface of planets.

In the case of gravitational fields, a **field line** gives the path that a smaller mass takes towards a larger mass due to the force of gravity.

Newton's law of gravitation

Gravity is the universal attractive force that acts between all matter.



Newton determined the **force of gravitation** between two masses to be:

This equation is in the data booklet.

$$F = \frac{Gm_1m_2}{r^2}$$

F = force due to gravity
 G = gravitational constant
 $= 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
 m_1 = mass of object 1
 m_2 = mass of object 2
 r = distance between centres of mass

So the force due to gravity

- increases with mass
- decreases with distance
- is proportional to G

Gravitational field strength

The gravitational field strength, g , is the force, F , per unit mass, m , on an object in a uniform gravitational field.

$$g = \frac{F}{m}$$

The gravitational field strength of the Earth near the surface is 9.81 N kg^{-1} .

The magnitude for g in a radial field is:

$$g = \frac{GM}{r^2}$$

Gravitational fields

Gravitational potential

Gravitational potential energy (E_p):

the energy possessed by an object due to its **place in a gravitational field**. The gravitational potential energy of an object will be zero at infinity as the effects of the gravitational field will be negligible at this distance.

Gravitational potential (V):

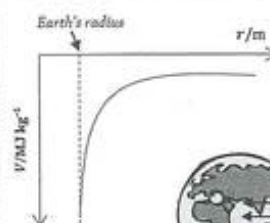
the gravitational potential energy per unit mass equal to the **work per unit mass to move an object from infinity to that point**. The gravitational potential is zero at infinity.

Gravitational potential in a radial field:

$$V = -\frac{GM}{r}$$

The **gravitational potential difference** is the difference in gravitational potential between two points in space:

$$\Delta V = V_2 - V_1$$



The potentials at points r_1 and r_2 are

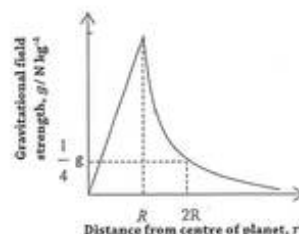
$$V_1 = -\frac{GM_{\text{Earth}}}{r_1} \text{ and } V_2 = -\frac{GM_{\text{Earth}}}{r_2}$$

To move an object (such as a rocket) from r_1 to r_2 , the work done would have to be $\Delta W = m_{\text{rocket}}(V_2 - V_1)$ or

$$\Delta W = m_{\text{rocket}} \left(-\frac{GM_{\text{Earth}}}{r_2} + \frac{GM_{\text{Earth}}}{r_1} \right)$$

$$g = -\frac{\Delta V}{\Delta r}$$

Therefore $-g$ = gradient of $V-r$ graph and ΔV is enclosed in a $g-r$ graph



The work done ΔW in moving a mass m from V_1 to V_2 is equal to the **change in its gravitational potential energy**:

$$\Delta W = m\Delta V$$

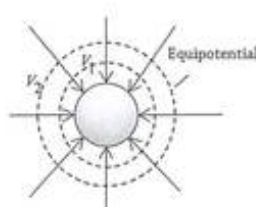
This means that a mass of 1 kg raised by 1 m from the Earth's surface gains 9.81 J of energy.

Equipotential surfaces

Equipotential surfaces are surfaces of **constant potential**.

When travelling along an equipotential the potential remains constant, there is no change in potential (ΔV). Therefore, since $\Delta W = m\Delta V$, then **no work is done when moving along an equipotential**.

Orbits represent equipotentials – planets do **no work** as they travel around the Sun.



Orbits of planets and satellites

3

Geostationary satellites

A **geostationary satellite** is a satellite that has an **orbital period** around Earth's equator of **one day** (24 hours).

Since the rotational period is also 24 hours, then the satellite stays at a **fixed position** above the Earth's equator.

The distance of a geostationary satellite above Earth can be found using **Kepler's third law** derived below.

Geostationary satellites are used in **communications** and **navigation systems**, such as the satnav that is used in cars.

A **synchronous orbit** refers to a specific orbit taken by a satellite where the orbital period is the **same as the rotational period** and in the **same direction** of rotation as the object it's orbiting.

Derivation of Kepler's third law

$$F_{\text{centripetal}} = F_{\text{gravitation}}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$\left(v = \frac{2\pi r}{T} \right)$$

$$\frac{(2\pi r)^2}{T^2} = \frac{GM}{r}$$

$$\frac{4\pi^2 r^3}{T^2} = GM$$

$$\frac{GM}{4\pi^2} = \text{constant}$$

$$T^2 \propto r^3$$

Escape velocity

Escape velocity is the minimum velocity required for an object to escape the gravitational field of a planet.

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}}$$



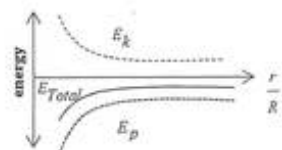
Energy of orbiting satellite

Total energy of satellite:

$$E_{\text{Total}} = E_p + E_k$$

$$E_{\text{Total}} = -\frac{GMm}{r} + \frac{GMm}{2r}$$

$$E_{\text{Total}} = -\frac{GMm}{2r}$$



The potential energy of the satellite dominates the total energy of the satellite – the total energy is negative and becomes constant at greater distances.