

Higher Topic: Fractions, Decimals, Percentages and Ratio 1

Topic/Skill	Definition/Tips	Example
Simplifying Fractions	Divide the numerator and denominator by the highest common factor.	$\frac{20}{45} = \frac{4}{9}$
Equivalent Fractions	Fractions which represent the same value .	$\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{60}{150} \text{ etc.}$
Changing between improper fractions and mixed numbers	Improper fraction to mixed number - divide numerator by denominator to get whole number. The remainder is the new numerator. Mixed number to improper fraction – multiply whole number by denominator and add the numerator. The denominator always stays the same.	$\frac{17}{5} \rightarrow 17 \div 5 = 3$ remainder 2. So the mixed number is $3\frac{2}{5}$ $4\frac{1}{3} \rightarrow 4 \times 3 + 1 = 13$. So the improper fraction is $\frac{13}{3}$
Comparing Fractions	To compare fractions, they each need to be rewritten so that they have a common denominator. Ascending means smallest to biggest. Descending means biggest to smallest.	Put in to ascending order : $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}, \frac{1}{2}$. Equivalent: $\frac{9}{12}, \frac{8}{12}, \frac{10}{12}, \frac{6}{12}$ Correct order: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
Adding or Subtracting Fractions	Find the LCM of the denominators to find a common denominator. Use equivalent fractions to change each fraction to the common denominator. Then just add or subtract the numerators and keep the denominator the same.	$\frac{2}{3} + \frac{4}{5}$ LCM of 3 and 5 = 15 $\frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$
Multiplying Fractions	Multiply the numerators together and multiply the denominators together. Then simplify .	$\frac{3}{8} \times \frac{2}{9} = \frac{6}{72} = \frac{1}{12}$
Dividing Fractions	‘Keep, Flip, Change – KFC’ Keep the first fraction the same Flip the second fraction upside down Change the divide to a multiply Multiply by the reciprocal of the second fraction.	$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$
Mixed number arithmetic	Use the same rules for adding, subtraction, multiplying , and dividing fractions but convert the mixed numbers to improper fractions first.	$2\frac{1}{3} + 1\frac{1}{5}$ $\frac{7}{3} + \frac{6}{5} = \frac{35}{15} + \frac{18}{15}$ $\frac{53}{15} = 3\frac{8}{15}$
Integers as fractions	All integers (whole numbers) can be written as fractions with denominator 1 .	$7 = \frac{7}{1} \quad 45 = \frac{45}{1}$
Fraction of an Amount	Divide by the bottom , times by the top	Find $\frac{2}{5}$ of £60 $60 \div 5 = 12$ $12 \times 2 = £24$

Reciprocal	The reciprocal of a number is 1 divided by the number. The reciprocal of x is $\frac{1}{x}$ When we multiply a number by its reciprocal we get 1. This is called the 'multiplicative inverse'.	The reciprocal of 5 is $\frac{1}{5}$ The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, because $\frac{2}{3} \times \frac{3}{2} = 1$
Fractions to Decimals	Divide the numerator by the denominator using the bus stop method.	$\frac{3}{8} = 3 \div 8 = 0.375$
Decimals to Fractions	Write as a fraction over 10, 100 or 1000 and simplify.	$0.36 = \frac{36}{100} = \frac{9}{25}$
Percentages to Decimals	Divide by 100	$8\% = 8 \div 100 = 0.08$
Decimals to Percentages	Multiply by 100	$0.4 = 0.4 \times 100\% = 40\%$
Fractions to Percentages	Percentage is just a fraction out of 100. Make the denominator 100 using equivalent fractions. When the denominator doesn't go in to 100, divide the numerator by the denominator and multiply by 100.	$\frac{3}{25} = \frac{12}{100} = 12\%$ $\frac{9}{16} = 9 \div 16 \times 100 = 56.25\%$
Percentages to Fractions	Percentage is just a fraction out of 100. Write the percentage over 100 and simplify.	$14\% = \frac{14}{100} = \frac{7}{50}$
Recurring decimals	A recurring decimal has a digit (or group of digits) that repeat forever. Dots are written above the first and last repeating digits.	$0.\dot{8} = 0.888888...$ $0.\dot{3}\dot{5} = 0.353535...$ $0.\dot{6}8\dot{7} = 0.687687...$
Recurring Decimal to Fraction	Set the decimal equal to x. Multiply x by a multiple of 10 until you have two x values with the same recurring decimal. Subtract the two x values from one another and divide by the number of x's left.	$x = 0.1212121212...$ $100x = 12.1212121212...$ $100x - x = 12.121212 - 0.121212$ $99x = 12$ $x = \frac{12}{99} = \frac{4}{33}$
Recurring or terminating?	To tell if a fraction is equivalent to a terminating or recurring decimal, find the prime factors of the denominator. If these are only 2's and 5's, the decimal will terminate. Otherwise it will be recurring.	$\frac{13}{40} \rightarrow$ denominator = $2 \times 2 \times 2 \times 5$, so will terminate (= 0.325) $\frac{13}{60} \rightarrow$ denominator = $2 \times 2 \times 3 \times 5$, so will be recurring (= 0.2166666666...)
Calculating percentages (non-calculator)	Break down into parts which are easily calculated – 10%, 5%, 1%, 50% etc	Find 23% of 360 10% of 360 = $360 \div 10 = 36 \rightarrow 20\% = 72$ 1% of 360 = $360 \div 100 = 3.6 \rightarrow 3\% = 10.8$ So 23% = $72 + 10.8 = 82.8$

Calculating percentages (calculator)	Change the percentage to a decimal or fraction and multiply .	Find 38% of 170 $0.38 \times 170 = 64.6$
One number as a percentage of another	Write as a fraction and then change to a percentage .	13 out of 20 = $\frac{13}{20} = \frac{65}{100} = 65\%$ 37 out of 40 = $\frac{37}{40} = 37 \div 40 \times 100 = 92.5\%$
Percentage Change	$\frac{\text{Difference}}{\text{Original}} \times 100\%$	A games console is bought for £200 and sold for £250. $\% \text{ change} = \frac{50}{200} \times 100 = 25\%$
Profit and loss	Profit and loss are both percentage changes. Profit is an increase . Loss is a decrease .	A car was bought for £1200 and sold for £840. $\% \text{ loss} = \frac{360}{1200} \times 100 = 30\% \text{ loss}$
Increase or Decrease by a Percentage	Non-calculator: Find the percentage and add or subtract it from the original amount. Calculator: Find the percentage multiplier and multiply.	<u>Increase 500 by 20% (Non Calc):</u> 10% of 500 = 50 so 20% of 500 = 100 500 + 100 = 600 <u>Decrease 800 by 17% (Calc):</u> 100% - 17% = 83% 83% $\div$ 100 = 0.83 0.83 $\times$ 800 = 664
Percentage Multiplier	The number you multiply a quantity by to increase or decrease it by a percentage . Add or subtract from 100% and change to a decimal.	The multiplier for increasing by 12% is 1.12 The multiplier for decreasing by 12% is 0.88 The multiplier for increasing by 100% is 2.
Reverse Percentage	Divide the amount by the percentage multiplier . Look out for words like ' before ' or ' original '	A jumper was priced at £48.60 after a 10% reduction. Find its original price. 100% - 10% = 90% = 0.9 $\pounds 48.60 \div 0.9 = \pounds 54$
Simple Interest	Interest calculated as a percentage of the original amount.	£1000 invested for 3 years at 10% simple interest. 10% of £1000 = £100 Interest = $3 \times \pounds 100 = \pounds 300$
Compound interest	Interest calculated as a percentage of the current amount. Find the multiplier for the increase/decrease and raise it to the power of the number of times this interest is applied. Then multiply the original value by this.	£1000 invested for 3 years at 10% compound interest. Multiplier for 10% increase is 1.1 $\pounds 1000 \times 1.1^3 = 1331$
Accuracy	Fractions are more accurate than rounded decimals or percentage equivalents.	$\frac{3}{7}$ is more accurate than 0.4286 (which has been rounded).

Ratio	Ratio compares the size of one part to another part . Written using the ':' symbol.	3 : 1 
Simplifying ratios	Divide all parts of the ratio by a common factor .	$5 : 10 = 1 : 2$ (divide both by 5) $14 : 21 = 2 : 3$ (divide both by 7)
Ratios in the form $1 : n$ or $n : 1$	Divide both parts of the ratio by one of the numbers to make one part equal 1 .	$5 : 7 = 1 : \frac{7}{5}$ in the form $1 : n$ $5 : 7 = \frac{5}{7} : 1$ in the form $n : 1$
Sharing in a ratio	1. Add the total parts of the ratio. 2. Divide the amount to be shared by this value to find the value of one part. 3. Multiply this value by each part of the ratio. Use only if you know the total .	Share £60 in the ratio 3 : 2 : 1. Add $3 + 2 + 1 = 6$ Divide $60 \div 6 = 10$ Multiply $3 \times 10 = 30$, $2 \times 10 = 20$, $1 \times 10 = 10$ £30 : £20 : £10
Ratio already shared	Find what one part of the ratio is worth using the unitary method .	Money was shared in the ratio 3 : 2 : 5 between Ann, Bob and Cat. Given that Bob had £16, find out the total amount of money shared. £16 = 2 parts, so £8 = 1 part $3 + 2 + 5 = 10$ parts, so $8 \times 10 = £80$
Combining Ratios	Make the ratio numbers for the common component the same.	The ratio of A:B is 3:2 and the ratio of B:C is 5:4. Make B = 10 in each, then A:B:C = 15 :10 :8
Ratio and scale	If the ratio for the scale factor is $1 : n$, to find the real length, multiply the scale length by n. To find the length on the scale model, divide the real length by n.	A scale of 1cm for every 2m means a ratio of $1 : 200$ A length of 3cm on the model represents 600cm or 6m in real life. A length of 10m in real life would be 5cm on the model.
Ratios as linear functions	Write the ratio as $x : y$ and decide what has to be done to make the parts equal.	The ratio of A to B is 3 : 5 $3 : 5 = x : y$ To make them equal we have to multiply x by 5 and y by 3. $5x = 3y$
Proportion	Proportion compares the size of one part to the size of the whole . Usually written as a fraction.	In a class with 13 boys and 9 girls, the proportion of boys is $\frac{13}{22}$ and the proportion of girls is $\frac{9}{22}$
Unitary method	Finding the value of a single unit and then find the necessary value by multiplying the single unit value.	3 cakes require 450g of sugar to make. Find how much sugar is needed to make 5 cakes. $3 \text{ cakes} = 450\text{g}$ So 1 cake = 150g ($\div$ by 3) So 5 cakes = 750g ($\times$ by 5)
Currency conversion	To change from pounds to a different currency, multiply by the exchange rate . Divide by exchange rate to get back to pounds.	If £1 = \$1.32, what is £600 worth? $600 \times 1.32 = \$792$
Direct proportion	If two quantities are in direct proportion, as one increases , the other increases by the same percentage.	For example, speed and distance travelled are directly proportional. Values will be in the same ratio.