

Topic: Algebra 1

Topic/Skill	Definition/Tips	Example
Key Vocabulary:		
Term	Either a single number or variable, or numbers and variables multiplied together	$4x$ or 7
Variable	A symbol , usually a letter , which represents a number which is usually unknown.	In the equation $x + 2 = 5$, x is the variable.
Expression	A mathematical statement written using symbols, numbers or letters	$3x + 2$ or $5y^2$
Equation	A statement showing that two expressions are equal	$2y - 17 = 15$
Identity	An equation that is true for all values of the variables An identity uses the symbol: $\equiv$	$2x \equiv x + x$
Formula	Shows the relationship between two or more variables	Area of a rectangle = length x width $A = L \times W$
Function	A special relationship where each input has a single output . It is often written as " f(x) " where x is the input value.	$f(x) = \frac{x}{2}$ ("f of x equals x divided by 2")
Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers
Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10
Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24
Coefficient	A number used to multiply a variable . It is the number that comes before/in front of a letter.	$6z$ 6 is the coefficient z is the variable
Odds and Evens	An even number is a multiple of 2 An odd number is an integer which is not a multiple of 2 .	If n is an integer (whole number): An even number can be represented by $2n$ or $2m$ etc. An odd number can be represented by $2n-1$ or $2n+1$ or $2m+1$ etc.
Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: $n, n+1, n+2$ etc. are consecutive integers.
Quadratic	A quadratic expression is of the form $ax^2 + bx + c$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions:

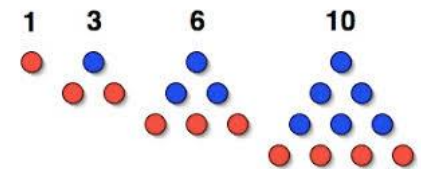
	where a, b and c are numbers, $a \neq 0$	$2x^3 - 5x^2$
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Simplifying Expressions	Collect 'like terms'. Be careful with negatives. x^2 and x are not like terms. x times x The answer is x^2 not $2x$. $p \times p \times p$ The answer is p^3 not $3p$ $p + p + p$ The answer is $3p$ not p^3 $4p^2 \times 3p^4$ Multiply the integers together, then use index laws for the powers: $4 \times 3 = 12$ $p^2 \times p^4 = p^6$	$2x + 3y + 4x - 5y + 3 = 6x - 2y + 3$ $3x + 4 - x^2 + 2x - 1 = 5x - x^2 + 3$ Squaring is multiplying by itself, not by 2. If $p=2$, then $p^3=2 \times 2 \times 2=8$, not $2 \times 3=6$ If $p=2$, then $2+2+2=6$, not $2^3 = 8$ $= 12p^6$
Substitution	Replace letters with numbers. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2$ and $c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
Substitute into formula	Including maths formula e.g $A = \pi r^2$ And other subjects e.g $v=u + at$ Remember that BIDMAS applies and being vigilant with negative numbers and powers	Radius = 4.5 cm, find the area $A = \pi r^2$ $A = \pi \times 4.5^2$
Simplify terms involving index laws	Including to the power of zero, negative powers and fraction powers	Fully simplify $\frac{9p^3 \times 4p^{-1/2}}{5p^{-1/2}},$ writing your answer in the form ap^b .
Expand	To expand a bracket, multiply each term in the bracket by the expression outside the bracket. If there is more than one bracket which are added /subtracted together, expand them separately and then simplify. Watch out for negatives here. -2×-2 becomes $+8$ when expanding the second bracket	$3(m + 7) = 3m + 21$ $3(m+7) - 2(m-4)$ $= 3m + 21 - 2m + 8$ Now collect like terms: $= 5m + 29$
Product of two linear expressions	When multiplying two linear expressions , every term in the first expression has to be multiplied by every term in the second expression.	$(y + 2)(y + 3)$ F: $y \times y = y^2$ O: $y \times 3 = 3y$ I: $2 \times y = 2y$ L: $2 \times 3 = 6$ $y^2 + 3y + 2y + 6 = y^2 + 5y + 6$

Factorise	The reverse of expanding . Factorising is writing an expression as a product of terms by ' taking out ' a common factor .	$6x - 15 = 3(2x - 5)$, where 3 is the common factor. $3x^2 - 12x$ Divide both terms by 3x to become: $3x(x - 4)$
Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that add to give b and multiply to give c .	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
Difference of Two Squares	An expression of the form $a^2 - b^2$ can be factorised to give $(a + b)(a - b)$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
Solve	To find the answer /value of something Use inverse operations on both sides of the equation (balancing method) until you find the value for the letter. Golden Rules: Always do the same to one side of the = as you do to the other Use the opposite operation to eliminate	Solve $2x - 3 = 7$ Add 3 on both sides $2x = 10$ Divide by 2 on both sides $x = 5$ Solve $2x^2 - 3 = 47$ Add 3 to both sides $2x^2 = 50$ Now divide by 2 $x^2 = 25$ Now Square root $x = 5$
Writing Formulae	Substitute letters for words in the question.	Bob charges £3 per window and a £5 call out charge.

		$C = 3N + 5$ Where N=number of windows and C=cost
Solving Quadratics ($ax^2 + bx = 0$)	Factorise and then solve = 0 .	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0$ or $x = 3$
Solving Quadratics by Factorising ($a = 1$)	Factorise the quadratic in the usual way. Solve = 0 Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5$ or $x = 2$
Solving Quadratics by Factorising ($a \neq 1$)	Factorise the quadratic in the usual way. Solve = 0 Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2}$ or $x = -4$
Completing the Square (when $a = 1$)	A quadratic in the form $x^2 + bx + c$ can be written in the form $(x + p)^2 + q$ 1. Write a set of brackets with x in and half the value of b . 2. Square the bracket. 3. Subtract $\left(\frac{b}{2}\right)^2$ and add c . 4. Simplify the expression. You can use the completing the square form to help find the maximum or minimum of quadratic graph.	Complete the square of $y = x^2 - 6x + 2$ Answer: $(x - 3)^2 - 3^2 + 2$ $= (x - 3)^2 - 7$ The minimum value of this expression occurs when $(x - 3)^2 = 0$, which occurs when $x = 3$ When $x = 3$, $y = 0 - 7 = -7$ Minimum point = $(3, -7)$
Completing the Square (when $a \neq 1$)	A quadratic in the form $ax^2 + bx + c$ can be written in the form $p(x + q)^2 + r$ Use the same method as above, but factorise out a at the start.	Complete the square of $4x^2 + 8x - 3$ Answer: $4[x^2 + 2x] - 3$ $= 4[(x + 1)^2 - 1^2] - 3$ $= 4(x + 1)^2 - 4 - 3$ $= 4(x + 1)^2 - 7$
Solving Quadratics by Completing the Square	Complete the square in the usual way and use inverse operations to solve .	Solve $x^2 + 8x + 1 = 0$ Answer: $(x + 4)^2 - 4^2 + 1 = 0$ $(x + 4)^2 - 15 = 0$ $(x + 4)^2 = 15$ $(x + 4) = \pm\sqrt{15}$ $x = -4 \pm \sqrt{15}$
Solving Quadratics using the	A quadratic in the form $ax^2 + bx + c = 0$ can be solved using the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Solve $3x^2 + x - 5 = 0$ Answer: $a = 3, b = 1, c = -5$

Quadratic Formula	Use the formula if the quadratic does not factorise easily.	$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 3 \times -5}}{2 \times 3}$ $x = \frac{-1 \pm \sqrt{61}}{6}$ $x = 1.14 \text{ or } -1.47 \text{ (2 d.p.)}$
Change the subject of an equation	Use inverse operations on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz + 1}{2} = x$ We now have x as the subject.
Change the subject including when the subject appears on both sides	Re-arrange to get all of the subject on one side of the = sign Factorize to isolate the subject Divide by the bracket to complete changing the subject	$4 - \frac{x}{7} = 7x - 8$ $7x - 8 = 4 - \frac{x}{7} \text{ (+8)}$ $7x = 12 - \frac{x}{7} \text{ (all terms by 7)}$ $49x = 84 - x \text{ (+x)}$ $50x = 84 \text{ (divide by 50)}$ $x = \frac{84}{50}$
Simultaneous Equations	A set of two or more equations, each involving two or more variables (letters). The solutions to simultaneous equations satisfy both/all of the equations.	$2x + y = 7$ $3x - y = 8$ $x = 3$ $y = 1$
Solving Simultaneous Equations (by Elimination)	Balance the coefficients of one of the variables. Eliminate this variable by adding or subtracting the equations (Same Sign Subtract, Different Sign Add) Solve the linear equation you get using the other variable. Substitute the value you found back into one of the previous equations. Solve the equation you get. Check that the two values you get satisfy both of the original equations.	$5x + 2y = 9$ $10x + 3y = 16$ Multiply the first equation by 2. $10x + 4y = 18$ $10x + 3y = 16$ Same Sign Subtract (+10x on both) $y = 2$ Substitute $y = 2$ in to equation. $5x + 2 \times 2 = 9$ $5x + 4 = 9$ $5x = 5$ $x = 1$ Solution: $x = 1, y = 2$

Solving Linear and Quadratic Simultaneous Equations	Method 1: If both equations are in the same form (eg. Both $y = \dots$): 1. Set the equations equal to each other. 2. Rearrange to make the equation equal to zero. 3. Solve the quadratic equation. 4. Substitute the values back in to one of the equations. Method 2: If the equations are not in the same form: 1. Rearrange the linear equation into the form $y = \dots$ or $x = \dots$ 2. Substitute in to the quadratic equation. 3. Rearrange to make the equation equal to zero. 4. Solve the quadratic equation. 5. Substitute the values back in to one of the equations. You should get two pairs of solutions (two values for x, two values for y.) Graphically, you should have two points of intersection.	<u>Example 1</u> Solve $y = x^2 - 2x - 5$ and $y = x - 1$ $x^2 - 2x - 5 = x - 1$ $x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$ $x = 4 \text{ and } x = -1$ $y = 4 - 1 = 3 \text{ and}$ $y = -1 - 1 = -2$ Answers: (4,3) and (-1,-2) <u>Example 2</u> Solve $x^2 + y^2 = 5$ and $x + y = 3$ $x = 3 - y$ $(3 - y)^2 + y^2 = 5$ $9 - 6y + y^2 + y^2 = 5$ $2y^2 - 6y + 4 = 0$ $y^2 - 3y + 2 = 0$ $(y - 1)(y - 2) = 0$ $y = 1 \text{ and } y = 2$ $x = 3 - 1 = 2 \text{ and } x = 3 - 2 = 1$ Answers: (2,1) and (1,2)
<u>Sequences</u>		
Linear Sequence	A number pattern with a common difference.	2, 5, 8, 11... is a linear sequence
Term	Each value in a sequence is called a term.	In the sequence 2, 5, 8, 11..., 8 is the third term of the sequence.
Term-to-term rule	A rule which allows you to find the next term in a sequence if you know the previous term.	First term is 2. Term-to-term rule is 'add 3' Sequence is: 2, 5, 8, 11...
nth term	A rule which allows you to calculate the term that is in the nth position of the sequence. Also known as the 'position-to-term' rule. n refers to the position of a term in a sequence.	nth term is $3n - 1$ The 100th term is $3 \times 100 - 1 = 299$

Finding the nth term of a linear sequence	1. Find the difference . 2. Multiply that by n . 3. Substitute $n = 1$ to find out what number you need to add or subtract to get the first number in the sequence .	Find the nth term of: 3, 7, 11, 15... 1. Difference is +4 2. Start with $4n$ 3. $4 \times 1 = 4$, so we need to subtract 1 to get 3. nth term = $4n - 1$
Fibonacci type sequences	A sequence where the next number is found by adding up the previous two terms	The Fibonacci sequence is: 1, 1, 2, 3, 5, 8, 13, 21, 34 ... An example of a Fibonacci-type sequence is: 4, 7, 11, 18, 29 ...
Geometric Sequence	A sequence of numbers where each term is found by multiplying the previous one by a number called the common ratio, r .	An example of a geometric sequence is: 2, 10, 50, 250 ... The common ratio is 5 Another example of a geometric sequence is: 81, -27, 9, -3, 1 ... The common ratio is $-\frac{1}{3}$
Quadratic Sequence	A sequence of numbers where the second difference is constant . A quadratic sequence will have a n^2 term.	
Triangular numbers	The sequence which comes from a pattern of dots that form a triangle. 1, 3, 6, 10, 15, 21 ...	
nth term of a geometric sequence	ar^{n-1} where a is the first term and r is the common ratio	The nth term of 2, 10, 50, 250 is $2 \times 5^{n-1}$

nth term of a quadratic sequence	1. Find the first and second differences. 2. Halve the second difference and multiply this by n^2. 3. Substitute $n = 1, 2, 3, 4 \dots$ into your expression so far. 4. Subtract this set of numbers from the corresponding terms in the sequence from the question. 5. Find the nth term of this set of numbers. 6. Combine the nth terms to find the overall nth term of the quadratic sequence. Substitute values in to check your nth term works for the sequence.	Find the nth term of: 4, 7, 14, 25, 40.. Answer: Second difference = +4 $\rightarrow$ nth term = $2n^2$ Sequence: 4, 7, 14, 25, 40 $2n^2$ 2, 8, 18, 32, 50 Difference: 2, -1, -4, -7, -10 Nth term of this set of numbers is $-3n + 5$ Overall nth term: $2n^2 - 3n + 5$
Where else is the Algebra topic used: Graphs Angles Measures Number Transformations		Links to other subjects: Computer programming Science DT