

FURTHER KINEMATICS

WHAT DO I NEED TO KNOW

1. To solve problems involving constant acceleration in 2 dimensions, use the SUVAT equations with vector components where $\mathbf{u}$ is the initial velocity
 $\mathbf{a}$ is the acceleration
 $\mathbf{v}$ is the velocity at time t (t is a scalar)
 $\mathbf{r}$ is the displacement at time t
2. To solve problems involving variable acceleration in 2 dimensions, use calculus with vectors by considering each function of time (the vector component) separately.
3. When integrating a vector for a variable acceleration problem, the constant of integration, c , will also be a vector.
4. To find constants of integration, look for initial conditions or boundary conditions.
5. Displacement, velocity & acceleration can be given using i - j notation, or as column vectors.

FORMULAE

The formula to find the position vector, $\mathbf{r}$, of a particle starting at position $\mathbf{r}_0$ that is moving with constant velocity $\mathbf{v}$ is

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$$

Constant acceleration vector equations:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

Calculus for variable acceleration:

Velocity, if displacement is a function of time:

$$\mathbf{v} = \frac{d\mathbf{s}}{dt}$$

$$\int (\mathbf{v}) dt = \mathbf{s}$$

Acceleration, if velocity is a function of time

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{s}}{dt^2}$$

$$\int (\mathbf{a}) dt = \mathbf{v}$$

DOT NOTATION & DIFFERENTIATING VECTORS

Dot notation is a shorthand for differentiation with respect to time.

$$\dot{x} = \frac{dx}{dt}$$

$$\dot{y} = \frac{dy}{dt}$$

$$\ddot{x} = \frac{d^2x}{dt^2}$$

$$\ddot{y} = \frac{d^2y}{dt^2}$$

To differentiate a vector quantity in the form $f(t)\mathbf{i} + g(t)\mathbf{j}$, differentiate each function of time separately.

If $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, then $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$ and $\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$